Fast Geometric Algorithms for Tomographic Nondestructive Evaluation

Peyman Milanfar
SRI International, Building 409-44
333 Ravenswood Avenue, Menlo Park, California 94025 U.S.A.
Phone: (415) 859-3745, Fax: (415) 859-6259
E-mail: milanfar@unix.sri.com

Abstract

In this paper we describe the development and application of a novel approach to fast nondestructive evaluation (NDE) via direct estimation of significant features from tomographic data without image reconstruction. Classically, the term tomographic has been used to refer only to X-ray, and perhaps magnetic resonance techniques, but in our framework we point out that it has been recently shown that synthetic aperture radar (SAR), ultrasound, and laser radar measurements can all be interpreted in terms of tomographic projections. Hence, the algorithms described in this paper will be widely applicable to a variety of modalities used for the inspection of civil structures.

In numerous situations of practical interest, such as the inspection of large civil structures, a full set of data with high signal-to-noise ratio is usually difficult, if not impossible, to obtain due to physical and sensor contraints. Therefore, the accurate reconstruction of an image of the medium is often difficult. Hence, much research work to date has been concentrated on improving image reconstruction for NDE applications. Yet, in most NDE applications, the reconstructed image is not the final product of interest. Instead, it is of interest to know whether a particular geometric feature (be it an anomalous crack or other object of interest) is present, and if so, to obtain a rough estimate of its shape and size. That is to say, a pixel-by-pixel reconstruction of the image is often unneccesary.

Our proposed method, based on a fundamental property of the (projection) Radon transform, is aimed at directly extracting geometric features from a set of given noisy, and possibly sparse data, without pixel-by-pixel image reconstruction. The propose method eliminates the need for full image reconstruction in arriving at the image features, hence resulting in significantly fewer overall computations. In addition, this approach allows us to compute easily the statistics for the estimated features, therefore resulting in easily quantifiable performance characteristics of the approach.

Several important advantages inherent to our proposed approach include 1) real-time processing, 2) process automation, and 3) improved performance. In this paper we will present examples of these advantages and point out possible extensions and generalizations of the proposed ideas.

1 Introduction

NDE is an important part of many industrial processes. NDE techniques and equipment have been developed to facilitate the important requirements of quality control and product assurance for a variety of purposes. In fact, the worldwide market for NDE equipment now exceeds $700 million and the equipment finds use in virtually all industries. In particular, with the advent of more sophisticated processors, measurement equipment, and displays, the value of 2-D NDE techniques such as radiographic and ultrasonic methods to the industry is on the rise. While much progress has recently been made in improving the overall speed and performance of these techniques, most are still far from being readily applicable to practical problems in the field. In particular, tomographic systems have their own specific shortcomings. The first of these drawbacks is that in most NDE applications, due to various physical constraints, tomographic data can only be collected over a narrow range of angles and with fairly poor quality (Tam, 1987). This sparse and noisy data are then typically used to reconstruct an image of the underlying field of interest. This reconstructed image is subsequently used to detect or estimate a flaw. The image reconstruction step in the overall tomographic NDE process is highly ill-posed even when complete data is available. This fact, combined with the poor quality and limited quantity of the data, make this step a weak point of tomographic NDE. We intend to mitigate this important difficulty by designing flexible algorithms that directly extract relevant features from the raw data set without image reconstruction. As a result of bypassing the direct reconstruction step, our proposed algorithms will also be computationally efficient. The extracted features will serve as sufficient statistics for detection and, in many instances, even suffice for the reconstruction of a *region of interest* in the image without pixel-by-pixel reconstruction of the entire field. The flexibility to choose the particular type of image features (e.g., a geometric moment) and the gross scale of the information they carry (e.g., the order of the computed moment), from the raw data, makes our proposed approach exceedingly useful. Most feature-extraction algorithms based on our approach will consist of linear estimation problems. As a result, not only is the statistical analysis of the quality of these estimated features rather simple, but the solution to the feature-extraction problem itself will be fast.

2 Overview

The objectives of NDE can, in many cases, be summarized as the solution to a combination of several problems. These problems are: 1) object detection, 2) object localization, 3) object characterization, and 4) object identification. As different as these problems may be for any particular application, they share a common denominator. In particular, in all four cases, a query is being made about the geometry of a medium. In those applications where the sensing modality allows for the reconstruction of an image of the medium (or a cross-section thereof), the reconstructed image has been typically used as the means of arriving at geometric information (Tam, 1987). In particular, the applications of tomographic techniques to problems in NDE have so far largely paralleled the approach taken in medical imaging. That is to say, to study a a non-biological sample, first the collected data are used to *reconstruct* an image of the sample. Next, an algorithm, or more typically a human expert, looks for significant features in the reconstructed image to detect, localize, or characterize a possible anomaly within the sample. In numerous situations of practical interest, however, a full set of data with high signal-to-noise ratio is difficult, if not impossible, to obtain. Therefore, the accurate reconstruction of the image is often difficult. Hence, much research work has been concentrated to date on improving image reconstruction from limited data. Yet, in most NDE applications, the reconstructed image is not the final product of interest. Instead, it is of interest to know whether a particular geometric feature (be it an anomaly or other object of interest) is present, and if so, to obtain a rough estimate of its shape and size. That is to say, a pixel-by-pixel reconstruction of the image is often unnecessary, if not inefficient and nonrobust.

Our proposed method is aimed at directly extracting features from a set of given noisy, and possibly sparse data, without pixel-by-pixel reconstruction. The proposed method is based on an interesting property of the Radon transform that relates specific features of the object being imaged directly to corresponding features of its projections. In particular, this property includes the celebrated Fourier Slice Theorem (Herman, 1980) as a special case. In fact, according to the basis functions chosen to extract these features from the data, these features may be a variety of different quantities such as geometric moments, Fourier coefficients, or wavelet scaling coefficients. When the moments of the image are sought after, a consequence of the aforementioned property is that "the k-th order moment of a projection of an image is equal to a linear combination of the k-th order moments of the image" (Milanfar, 1993).

3 Feature Extraction

Referring to Figure 1, the (projection) Radon transform $g(t,\theta)$ of a function $f(x,y)$, with support in the unit disk D, is defined for each pair (t,θ) as the integral of f over a line at angle $\theta + \frac{\pi}{2}$ with the x-axis and at radial distance t away from the origin. An elementary result (Helgason, 1980), which readily follows from the definition of the Radon transform, states that if $F(t)$ is *any* square integrable function on $[-1,1]$, then the following relation holds true:

$$\int_{-1}^{1} g(t,\theta)F(t)dt = \int\int_{D} f(x,y)F(x\cos(\theta) + y\sin(\theta))\, dx\, dy. \quad (1)$$

In plain terms, the above property states that the inner-product of $g(t,\theta)$ with $F(t)$ is exactly equal to the inner-product of the image $f(x,y)$ with F evaluated along the line $t = x\cos(\theta) + y\sin(\theta)$. Note that in the special case where $F(t)$ is a complex exponential, the celebrated Fourier Slice Theorem (Herman, 1980) is obtained that relates the Fourier transform of f to the Fourier transform of its projections. In keeping with the spirit of this well-known result, we shall henceforth refer to the more general result (1) as the Inner-Product Slice Theorem (IPST).

While the IPST has been well-known for at least three decades in the mathematics community, except for the special case of the Fourier Slice Theorem, little use of it has been made to date in the engineering community. Recent work (Milanfar et al., 1992; Milanfar, 1993; Milanfar et al., 1994; Milanfar et al., 1995; Milanfar et al., 1996) has concentrated on the direct extraction of geometric information from tomographic data via the estimation of moments. This can be accomplished by replacing $F(t)$ with the functions t^k for various k. For instance, letting $F(t) = t^0 = 1$, we see that

$$m_0 = \int_{-1}^{1} g(t,\theta)dt = \int\int_{D} f(x,y)dxdy = M_{0,0}. \quad (2)$$

Similarly, letting $F(t) = t$, we obtain

$$\begin{aligned} m_1(\theta) &= \int_{-1}^{1} g(t,\theta) t dt && (3) \\ &= \int\int_D f(x,y) x dx dy \cos(\theta) + \int\int_D f(x,y) y dx dy \sin(\theta) && (4) \\ &= M_{1,0}\cos(\theta) + M_{0,1}\sin(\theta) && (5) \end{aligned}$$

Normalizing both sides by (2), we get

$$\frac{m_1(\theta)}{m_0} = \frac{M_{1,0}}{M_{0,0}}\cos(\theta) + \frac{M_{0,1}}{M_{0,0}}\sin(\theta), \quad (6)$$

which demonstrates that the center of mass of a projection at angle θ is a linear combination of the coordinates of the center of mass of the original object f with the coefficients being sinusoidal functions of the projection angle. Hence, given two projections at distinct angles, we can compute $m_1(\theta)$ at both angles and a linear system of two equations is obtained in the two unknowns $M_{1,0}$ and $M_{0,1}$, which can be readily solved. That is to say, the center of mass of f is obtained directly from two projections without image reconstruction.

More generally, we have shown that "m projections of an image from distinct viewing angles uniquely determine the first m moments of the image." In particular, this means that given m *noisy* projections of an image, estimates of the first m moments of the image can be uniquely obtained by solving a *linear* least-squares problem (Milanfar et al., 1992; Milanfar, 1993; Milanfar et al., 1994; Milanfar et al., 1995; Milanfar et al., 1996). More generally, $F(t)$ can be replaced by any member of a family of polynomials which span square integrable functions over the interval $[-1, 1]$ to yield the moments of the image with respect to that family. In particular, this includes the orthogonal families of Legendre and Tchebyshev polynomials.

4 Anomaly Detection

The detection or classification performance of a feature-based NDE algorithm will depend on the order and type of features used. An appropriate performance criterion can be defined and optimized to yield the best results. The definition and optimization of such a criterion can be based on either stochastic principles, such as the minimum description length (MDL) criterion (Rissanen, 1989) for determining the optimal number of features to use, or deterministic requirements such as computational efficiency, well-posedness and numerical conditioning of the feature extraction problem. To be concrete, consider the following simple scenario. Assume that given noisy projection data, an estimate of a vector of n geometric moments M_n of the underlying image is obtained. Under Gaussian white noise assumptions, the maximum likelihood (ML) estimate of this vector $\widehat{M}_n$ will be a Gaussian random vector with mean M_n and covariance Q_n. To use this feature vector to perform a test of two hypotheses H_0 and H_1, corresponding to the absence or presence of a flaw, respectively, the ML decision rule simplifies to the following:

$$(\widehat{M}_n - M_n^1)^T Q_n^{-1} (\widehat{M}_n - M_n^1) - (\widehat{M}_n - M_n^0)^T Q_n^{-1} (\widehat{M}_n - M_n^0) \left\{ \begin{array}{ll} > \gamma & \text{choose} \quad H_1, \\ < \gamma & \text{choose} \quad H_0 \end{array} \right\}, \quad (7)$$

where γ is the detection threshold, and M_n^1 and M_n^0 are the hypothesized moments of the object with and without a flaw, respectively.

In the above framework, the number of estimated moments n was held fixed. However, to adapt the number of moments for best performance, we can envision optimizing the detection probability as a function of the number of estimated moments for a fixed false-alarm rate.

5 Computational Complexity

As the proposed algorithm bypasses the image reconstruction step, we can expect significant computational savings as compared to existing techniques that rely on pixel-by-pixel reconstruction followed by feature extraction. As an example of what we might expect, assume that we wish to compute the geometric moments of a 256×256 image given 20 projections at uniformly spaced angles in the range $[0, \pi]$. Figure 2 shows two curves. In one, the number of floating point operations (FLOPs) performed is shown for the case where the image is first reconstructed using filtered-backprojection (Herman, 1980) and the moments are then computed from this reconstructed image. The second curve shows the number of FLOPs performed when the moments are estimated directly from the projection data. These curves were generated by explicitly carrying out the reconstruction and moment estimation steps for both approaches and counting the number of FLOPs. For this example, the computational savings are seen to be significant. We expect this to be the case in general.

6 Applications

In addition to the standard radiographic probes such as X-ray, the proposed approach is broadly applicable to a rich class of other sensor outputs. In particular, spotlight-mode SAR has an exact interpretation as a tomographic probe (Desai and Jenkins, 1992). Electro-optical sensors such as laser radar also have tomographic interpretations (Knight et al., 1989). Under suitable conditions, ultrasonic probes can be treated as tomographic probes as well (Kak, 1979). In fact, for the case of diffracting sources such as ultrasound and electromagnetic radiation, if the incident wave has wavelength that is sufficiently small compared to the size of the anomaly being sought, the Fourier Diffraction Theorem (Kak and Slaney, 1988) for weakly scattering objects is a very close approximation of the Fourier Slice Theorem hence allowing us to model the forward- or back-scatter measurements as transmission measurements along approximately straight lines. While this approximation may not be adequate for the purpose of direct imaging, in many scenarios it may well be suitable for feature extraction. For instance (Kak and Slaney, 1988), a hypothetical ultrasound experiment conducted at 10 MHz corresponds to a wavelength (in water) of 0.15 mm (Kak and Slaney, 1988). Measurements of any feature with size on the order of 15 mm or larger could be modeled as a straight line integral. Hence, IPST would apply and features could be directly extracted from raw data.

7 Summary/Conclusions

We have presented a general framework for the detection/extraction of object features/anomalies directly from tomographic probes without image reconstruction. We provided a brief description of a statistically based anomaly detection scheme based on the extracted features and also provided an example of how our proposed technique can offer significant computational savings over existing techniques. We described the wide array of applications that our proposed technique might have and indicated that the utility of the proposed technique reaches beyond that of transmission tomographic probes such as X-rays.

References

Desai, M. and W. K. Jenkins. Convolution backprojection image reconstruction for spotlight mode synthetic aperture radar. *IEEE Trans. on Image Processing*, 1(4):505–517, October 1992.

Helgason, Sigurdur. *Radon Transform*. Birkhauser, Boston, 1980.

Herman, G. T. *Image Reconstruction From Projections*. Academic Press, New York, 1980.

Kak, Avinash. Computerized tomography with x-ray, emission, and ultrasound sources. *Proceedings of the IEEE*, 67(9):1245–1271, September 1979.

Knight, F. K., S. R. Kulkarni, R. M. Marino, and J. K. Parker. Tomographic techniques applied to laser radar reflective measurements. *The Lincoln Laboratory Journal*, 2(2):143–160, Summer 1989.

Kak, Avinash and Malcolm Slaney. *Principles of Computerized Tomographic Imaging*. IEEE Press, 1988.

Milanfar, Peyman. *Geometric Estimation and Reconstruction From Tomographic Data*. PhD thesis, M.I.T., Department of Electrical Engineering, June 1993.

Milanfar, Peyman , William C. Karl, and Alan S. Willsky. Recovering the moments of a function from its radon-transform projections: Necessary and sufficient conditions. LIDS Technical Report LIDS-P-2113, MIT, Laboratory for Information and Decision Systems, June 1992.

Milanfar, Peyman, William C. Karl, and Alan S. Willsky. Reconstructing binary polygonal objects from projections: A statistical view. *CVGIP: Graphical Models and Image Processing*, 56(5):371–391, September 1994.

Milanfar, Peyman, William C. Karl, and Alan S. Willsky. A moment-based variational approach to tomographic reconstruction. *IEEE Trans. on Image Processing*, January 1996. In print.

Milanfar, Peyman, George C. Verghese, William C. Karl, and Alan S. Willsky. Reconstructing polygons from moments with connections to array processing. *IEEE Trans. on Signal Processing*, 43(2), February 1995.

Rissanen, Jorma. *Stochastic Complexity in Statistical Inquiry*, volume 15 of *Series in Computer Science*. World Scientific, 1989.

Tam, K. C. Limited-angle image reconstruction in non-destructive evaluation. C. H. Chen, editor, in *Signal processing and pattern recognition in nondestructive evaluation of materials*, volume 44 of *NATO ASI Series F: Computer and Systems Sciences*. Springer-Verlag, August 1987.

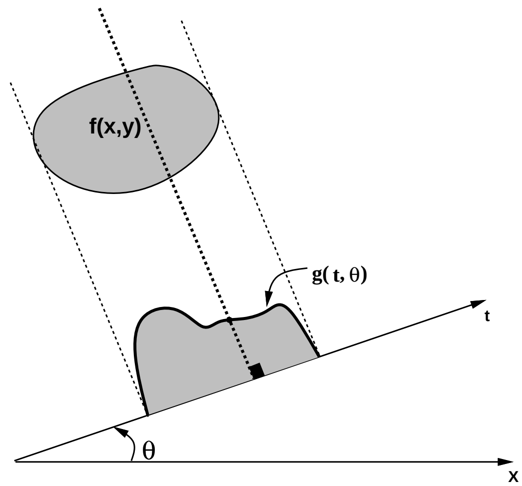

Figure 1: The Radon (projection) transform

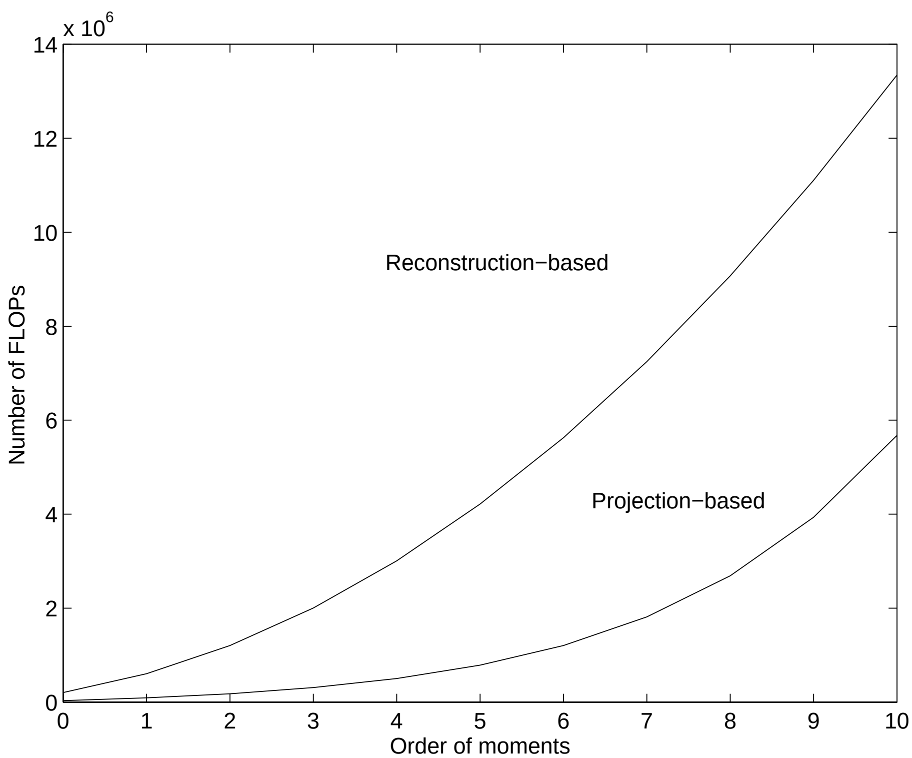

Figure 2: Example of computational savings with projection-based approach